$$y_n = \sum_m c_{nm} f_m$$

$$(f_l, y_n) = \sum_m c_{nm} (f_l, f_m)$$

$$c_{nl} = (f_l, y_n)$$

$$H_0 f_n(x) = \Omega \omega_{0_n}^2 f_n(x)$$

$$E \frac{d^2}{dx^2} \left(W T^3 \frac{d^2 y_n}{dx^2} \right) = -\rho W T \frac{d^2 y_n}{dt^2}$$

Tension (T) is constant, so it can be divided out:

$$E \frac{d^2}{dx^2} \left(W \frac{d^2 y}{dx^2} \right) = -\frac{\rho W}{T^2} \frac{d^2 y}{dt^2}$$

The derivatives need to be distributed, because Width (W) is dependent on x:

$$E \frac{d}{dx} \left(\frac{dW}{dx} \frac{d^2 y}{dx^2} + W \frac{d^3 y}{dx^3} \right) = \frac{\rho W}{T^2} \omega^2 y$$

$$\frac{d^2 W}{dx^2} \frac{d^2 y}{dx^2} + 2 \frac{dW}{dx} \frac{d^3 y}{dx^3} + W \frac{d^4 y}{dx^4} = \frac{\rho W}{E T^2} \omega^2 y$$

$$\Omega = \frac{\rho}{E T^2}$$

The matrix of the fourth derivative, H_o is symmetric and Hermitian. Therefore all its eigenfunctions are orthogonal.

$$H_o = \frac{d^4}{dx^4}$$

$$W H_o y + 2 W' y''' + W'' y'' = W \Omega \omega^2 y$$

$$H_o y + 2 \frac{W' y'''}{W} + \frac{W'' y''}{W} = \Omega \omega^2 y$$

$$H_o y + \frac{1}{W} \left(2 W' y''' + W'' y'' \right) = \Omega \omega^2 y$$

The rest of the expression can be notated as H', a non-symmetric matrix.

$$H' = 2 W' \frac{d^3}{dx^3} + W'' \frac{d^2}{dx^2}$$

$$H_o y + \frac{1}{W} H' y - \Omega \omega^2 y = 0$$

Given the values of Ω and ω, it's now possible to solve the homogenous equation for the coefficients of the linear combination of functions which define y.