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Energy calibration of the Radphi Lead Glass Detector

Richard Jones

University of Connecticut, Storrs CT

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Abstract

The Radphi experiment requires an energy calibration procedure for the lead glass calorimeter. In the absence of an available source of mono-energetic electrons or photons, the gains of the individual blocks as well as the overall energy scale must be determined based only on characteristic peaks that appear in the invariant mass spectra from reconstructed showers. A procedure is described that reliably extracts absolute gain coefficients for all 620 instrumented blocks from a sample containing of order 10^5 events. These events are a subset of the standard physics trigger, allowing redundant checks and time-dependent calibration to be carried out after the data are already collected.

The LGD calibration procedure is based upon the fact that when the positions and energies of individual showers in the calorimeter are combined to form invariant an mass, the mass spectrum exhibits prominent peaks corresponding to known mesons decaying into $n\gamma$ final states, eg. π^0, η, η', ω. These peaks are visible in the reconstructed spectra even before the gain calibration has been carried out, with individual tubes varying in gain by a factor of 2 or more. The gains were set during the experimental run by adjusting the high voltage on individual tubes until their responses to an injected light pulse from the calibration laser were approximately equalized. The pulser equalization procedure was repeated periodically throughout the run to take into account changes in the response of individual blocks arising from radiation damage and other sources of long-term drift during the experiment. Inhomogeneities in the light distribution led to physical gains on particular channels that differed by more than a factor of two from the mean, with a 25% r.m.s. deviation. The goal of the offline calibration is to measure these gain factors using experimental data, so that they can be used in turn to correct the data during reconstruction.

A calibration procedure based solely upon the reconstructed masses of known mesons is apparently problematic because it seeks to exploit one constraint (the known mass of the meson) to estimate multiple parameters (gain correction factors for every channel that contributes energy to the event). When many events containing a contribution from a given block are superimposed, however, the dependence of the average reconstructed mass on the gains of other blocks tends to wash out, leaving a bias that comes from the gain of the block itself. Removing this bias by applying a gain correction to this channel leads to a narrower peak in the mass plot. This is accomplished quantitatively by adjusting individual gain factors to optimize a single global function of the data, shown in Eq. 1.

$$F = \sum_{i=1}^{N} (m_i^2 - m_0^2)^2 + 2\lambda \sum_{i=1}^{N} (m_i^2 - m_0^2) \tag{1}$$

where N is the number of events in the calibration data sample and i denotes a single event in that sample. The masses m_0 and m_i are the physical mass of the meson being used for the calibration and the reconstructed mass in the LGD for event i. The first term in F measures the width of the reconstructed mass peak, while the second term is introduced with the Lagrange multiplier λ to embody the constraint $< m_i^2 >= m_0^2$.

For the purposes of Radphi, the most convenient meson for calibration turned out to be the π^0 which appears as the dominant structure in the 2γ invariant mass plot for 2-cluster events. All events that were reconstructed with exactly two clusters and whose invariant mass lay within $\pm 30\%$ of the center of the π^0 peak were candidates for the calibration sample. The 2γ invariant mass is given by Eq. 2.

$$m_i^2 = 2 p_{1i}\, p_{2i}\, (1 - \cos\gamma_i) \tag{2}$$

where p_{1i}, p_{2i} are the reconstructed energies of the two showers and γ_i is the angle between the centers of the two showers as viewed from the target.

The reconstructed energy p_{ji} is approximately equal to the observed energy s_{ji} in shower j, but contains additional nonlinear corrections to account for angle-dependent attenuation and output coupling effects.

$$p_{ji} = (1+g) \sum_{k \in S_j} E_{ki} = (1+g) s_{ji} \tag{3}$$

where k labels an individual block contributing to shower j. The nonlinearity correction factor g is weakly dependent on the observed shower energy s_{ji} but not on the block energies E_{ki} individually. A calibration step consists of introducing a small channel-dependent gain correction factor ϵ_k such that $E_{ki} \to E'_{ki} = (1+\epsilon_k) E_{ki}$, where the prime is used to denote the corresponding quantity after the gain correction is applied.

Minimizing F in Eq. 1 directly with respect to the variables ϵ_k is made difficult by the nonlinear dependence of m_i on the block energies that appears in the factor g in Eq. 3 and also in γ_i. Progress can be made by observing that small shifts in the gain of a single channel will have little effect on the nonlinear correction factors or the shower centroids, but will rescale the p_{ji} of its shower as shown in Eq. 4.

$$\frac{\partial {p'}_{ji}}{\partial \epsilon_k} \simeq p_{ji} \frac{E_{ki}}{s_j} \tag{4}$$

$$\frac{\partial {m'}_i^2}{\partial \epsilon_k} \simeq m_i^2 \frac{E_{ki}}{s_j} \tag{5}$$

These approximations lead to a linear equation which is satisfied at the minimum of the penalty function F.

$$\begin{aligned}
\frac{\partial F'}{\partial \epsilon_k} &= 2\sum_{i=1}^{N}({m'}_i^2 - m_0^2)\frac{\partial {m'}_i^2}{\partial \epsilon_k} + 2\lambda\sum_{i=1}^{N}\frac{\partial {m'}_i^2}{\partial \epsilon_k} \\
&= 2\sum_{i=1}^{N}\left(m_i^2 - m_0^2 + \lambda + \sum_{k'}\epsilon_{k'}\frac{\partial {m'}_i^2}{\partial \epsilon_{k'}}\right)\frac{\partial {m'}_i^2}{\partial \epsilon_k} \\
&= 0
\end{aligned} \tag{6}$$

The solution is given by

$$\epsilon_k = [C^{-1}]_{kk'}(D - \lambda L)_{k'} \tag{7}$$

where

$$\begin{aligned}
C_{kk'} &= \sum_{i=1}^{N}\left(\frac{\partial {m'}_i^2}{\partial \epsilon_k}\frac{\partial {m'}_i^2}{\partial \epsilon_{k'}}\right) \\
D_k &= -\sum_{i=1}^{N}\left((m_i^2 - m_0^2)\frac{\partial {m'}_i^2}{\partial \epsilon_k}\right) \\
L_k &= \sum_{i=1}^{N}\frac{\partial {m'}_i^2}{\partial \epsilon_k}
\end{aligned}$$

and the value of λ is fixed by the condition that the centroid of the reconstructed mass peak must lie at the physical mass.

$$\lambda = \frac{B + L^T C^{-1} D}{L^T C^{-1} L} \tag{8}$$

where B is the mass bias $\sum_{i=1}^{N}(m_i^2 - m_0)^2$. Starting off with a nominal gain factor of unity for all channels, gain corrections are calculated using Eq. 7 and applied iteratively until the procedure converges to $\epsilon_k \to 0$ for all k. In practice, it was found that special care must be taken in the way that the matrix C is inverted. C is a square symmetric matrix with 620 rows and columns whose elements are determined statistically by sampling a finite sample of N calibration events. Even for very large samples there are instabilities that appear when taking the inverse C^{-1} which demand careful treatment. The nature of these instabilities can be best understood by expressing C^{-1} in terms of its spectral decomposition Eq. 9.

$$[C^{-1}]_{kk'} = \sum_{\alpha}\frac{1}{c(\alpha)}e_k(\alpha)e_{k'}(\alpha) \tag{9}$$

where $c(\alpha)$ are the eigenvalues and $e(\alpha)$ the corresponding orthonormalized eigenvectors of C. Of the 620 eigenvalues of C, there are generally a few whose values are very small and statistically consistent with zero. These terms tend to dominate the behavior of C^{-1} if it is calculated using exact methods. A better approach instead is to truncate Eq. 9 and include only eigenvectors in the sum whose eigenvalues are adequately determined by the data. This truncation implicitly recognizes that there are some linear combinations of the gains which cannot be determined from the given data sample, and simply leaves them unchanged from the initial conditions. It is not the same as fixing any subset of the ϵ_k values to zero on a given step, which does not occur when this method is followed. Empirically it was found that good convergence was obtained after 8-10 iterations of the above procedure.