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Energy nonlinearity and depth correction of showers in the LGD

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Abstract

Prior work on energy calibration of the Radphi lead-glass calorimeter showed that the total light yield in an electromagnetic shower rises somewhat faster than linearly with shower energy. This has been explained as a consequence of the dependence of the average shower depth on energy, coupled with light attenuation in the glass block. Energy loss through leakage out the back of the block is also present, but would cause the light yield to rise slower than linearly with shower energy. At angles below 20^0 where showers see over 45cm of glass, it is plausible that the attenuation effect would win out over leakage. But above 20^0 (almost 50% of the solid angle subtended by the LGD is above 20^0) the effective thickness of the glass decreases, and somewhere one would expect the sign of the nonlinear effect to reverse. Thus the nonlinear energy correction must be a strong function of the polar angle of the shower. The same is true of the z-centroid of the shower in the glass. This report gives the results of a detailed Monte Carlo simulation that reproduces the known nonlinear behavior of the LGD for normal-incidence showers and provides the information required to carry out this correction at larger angles where the shower leakage is substantial.

1 Prior work

The total Čerenkov light yield in an electromagnetic shower measures to within a fraction of one percent the total $e^{\pm}$ track length in the shower. However only a small fraction of the total light generated actually reaches the photomultiplier tube, and that fraction depends on the shower position. Variations in the collection efficiency with where the shower enters the front face of a block are simply folded into the energy resolution of the counter because they are uncorrelated with the shower energy. But variations in the shower depth centroid are correlated with shower energy, and produce a nonlinearity in the measured photoelectron yield with shower energy. It is well known that the mean depth of a shower increases as the log of the shower energy [1]. Using a simple exponential attenuation model for the loss of Čerenkov light propagating down a block to the phototube, S. Teige showed [2] that the measured photoelectron yield vs shower energy is a power law.

$$p_e = E^{1+\epsilon} \tag{1}$$

where ϵ is of order a few percent. More recently D. Armstrong has shown that without such a correction one cannot obtain consistent values for the masses of the π^0 and η mesons independent of photon multiplicity in the event.

Attenuation is not the only nonlinear effect present. Another major effect that prior studies have not taken into account explicitly is shower leakage from loss of particles out the back of the LGD. This effect is also dependent on shower energy, but it enters with the opposite sign from attenuation: instead of increasing the gain with higher gamma energy it decreases it. S. Teige has pointed out in discussion that the competition between these two effects can be exploited to optimize the energy resolution at a certain block thickness. He indicated that the Radphi LGD blocks are somewhat longer than optimal for Radphi energies. This means that somewhere beyond 20^0, where showers begin to leak out the sides of the detector, one should observe the energy resolution first improving and then getting worse as shower leakage begins to dominate over attenuation.

In a prior study [3] I proposed a formula to be used in estimating the mean depth of showers inside the LGD, based on a knowledge of the shower energy E. This, in turn, feeds back to the calculation of the polar angle θ of the photon. Unfortunately this can only work reliably in the region $\theta < 20^0$ where shower leakage effects are small. At larger values of θ a more complicated procedure is required because there the centroid z of the observed shower is a function both of E and θ. Even worse, E itself is not known directly in this region either because it depends on a shower leakage correction that must depend strongly on θ. So we start off with measured values p_e and cluster coordinates (x, y) and we want to end up with corrected values for (θ, ϕ) and E. This system can be solved by iteration, by seeking the solution to the following three constraints.

$$\theta = \Theta(x, y, z) \tag{2}$$

$$z = Z(\theta, E) \tag{3}$$

$$p_e = L(\theta, E) \tag{4}$$

where $\Theta(x, y, z) = \arctan \sqrt{x^2 + y^2}/z$. The functions $Z(\theta, E)$ and $L(\theta, E)$ must be obtained from Monte Carlo simulation. The function $Z(\theta, E)$ should approach the form given in Eq. 3 of Ref. [3] for small values of θ. The function $L(\theta, E)$ should approach Eq. 1 for small θ. If the simulation passes both of these checks without fiddling then one may have confidence in its reliability in the region of large θ.

2 Simulations

In Ref. [3] a detailed Monte Carlo simulation of the LGD was described, based upon GEANT-3.21. The simulation tracks all shower particles down to a kinetic energy of 100KeV, where they stop emitting Čerenkov light and are stopped. The Čerenkov light is then propagated down the block, undergoing reflections until it is either absorbed or exits the block and is absorbed. Just downstream of the block is a phototube; those Čerenkov photons that are transmit into the glass surface of the phototube are evaluated according to the quantum efficiency of the tube at the given wavelength, and either discarded or counted in the photoelectron sum p_e. The measured attenuation spectrum of clean lead glass is used by the simulation inside the block. Rays that strike the surface of the block are stochastically transmitted or reflected according to the familiar Fresnel formulae.

The latter requires the knowledge of how the blocks are wrapped. In Ref. [3] I assumed that the block is surrounded by a layer of air followed by a non-reflecting surface. This permits total internal reflection at large incidence angles, but discards all photons that exit the glass. Under these conditions it was seen that the mean emission depth of the light observed at the phototube is not appreciably shifted by the presence of attenuation.

To check the sensitivity of this result to the wrapping, this study was repeated [4] with an alternative wrapping model. In this second model, the air gap was removed and a reflecting surface put in its place with a reflectivity of 90%. As described in that report, this change did make a noticeable difference, shifting the effective shower centroid deeper into the glass by about 2cm, independent of shower energy. Believing that this second model is a better approximation to the actual LGD than the former, I update Eq. 3 of Ref. [3] to read

$$z_0 = 102.9 + X_0 \left[ln\left(\frac{E}{E_c}\right) + 3.0 \right] \tag{5}$$

where X_0 is the radiation length of lead glass, about 3.1cm, and E_c is the critical energy of lead glass, about 14.5MeV. The 0 subscript in Eq. 5 is there to remind the reader that this equation describes showers at normal incidence to the glass.

To go beyond the restriction of normal incidence, a new simulation was performed. In this simulation the reflective wrapping model was used for the LGD. Photons were generated uniformly in energy from 0 to 2GeV. Runs of 1000 showers each were generated starting at $\theta = 1^0$ up to $\theta = 28^0$ in steps of 1^0. Each gamma ray was independently tracked from inside the target to when all of its energy was either absorbed or it exited the alcove area. All phototubes were summed for each event, i.e. there was no clustering performed, but only the blocks right around the shower core contributed appreciably to the p_e sum. For each detected Čerenkov photon a record was kept of the z coordinate where it originated, for purposes of calculating the third coordinate of the shower centroid.

The showers were then binned in θ and E, and the quantity p_e and the mean z were histogrammed on this grid. The results can be seen in Figs. 1-2. The error bars in these figures represent the r.m.s. fluctuations in those quantities from one shower to the next; the error on the mean value itself is about a factor of 30 smaller.

In Fig. 1 one sees at all energies the expected decrease in the observed yield that appears beyond $\theta = 20^0$. There are a couple of interesting things to note here. First, the transition from rising to falling gain does not show a kink, as would be obtained if one just calculated the geometrical wall thickness along the photon ray. This is because showers fan out as they develop and are not confined to the ray of the original photon. Even more interesting is the observation that the best energy resolution is obtained in a narrow angular band around $\theta = 22^0$. This is seen in the figure at all photon energies: the shortest %-error is obtained in the region just beyond the turnover. This is the effect described by S. Teige (see previous section) where at a certain block length there is a magic point where the effects of attenuation and shower leakage tend to cancel and shower conversion depth fluctuations do not affect the observed yield to leading order. It is interesting to see how dramatic that effect is in the simulation results.

The corresponding results for z are shown in Fig. 2. Here the decrease at 20^0 is less abrupt, but the effect is clearly visible. One might be surprised to see that the mean depth does not equal the front face of the glass at 28^0. If one looks at the Čerenkov z distribution for these showers, one sees a sharp peak near the front of the glass corresponding to the shower core, but then a tail extending downstream into the glass. This tail comes from shower fragments that happen to split off the core up near the front and move down into the block. As can be seen from Fig. 1, only a fragment of the shower is visible to the detector so there is a bias towards the part you can see.

Overall these results are quite encouraging. They show that we obtain a fairly good energy resolution all the way out to 25^0 and perhaps further. However to make effective use of the these high-angle showers we need to apply a θ-dependent gain correction. The fits shown by the curves in Figs. 1-2 are parameterizations of these two-dimensional functions that will enable us to apply this correction to our data.

3 Parameterizations

From the simulation data presented in the last section we can now extract approximate expressions for the functions $Z(\theta, E)$ and $L(\theta, E)$ of Eqs. 3-4. To carry out the fit for $L(\theta, E)$, the Monte Carlo data for p_e were histogrammed in bins of 200MeV $\times$ 1^0. For each energy bin there is an independent histogram of p_e vs θ, as shown in Fig. 1. Each of these was then submitted to fit to a 4-parameter formula consisting of a monopole plus a linear term, as follows.

$$L(\theta) = (\theta_0 - \theta)\left(a + \frac{b}{(\theta_0 - \theta)^2 + \delta^2} \right) \tag{6}$$

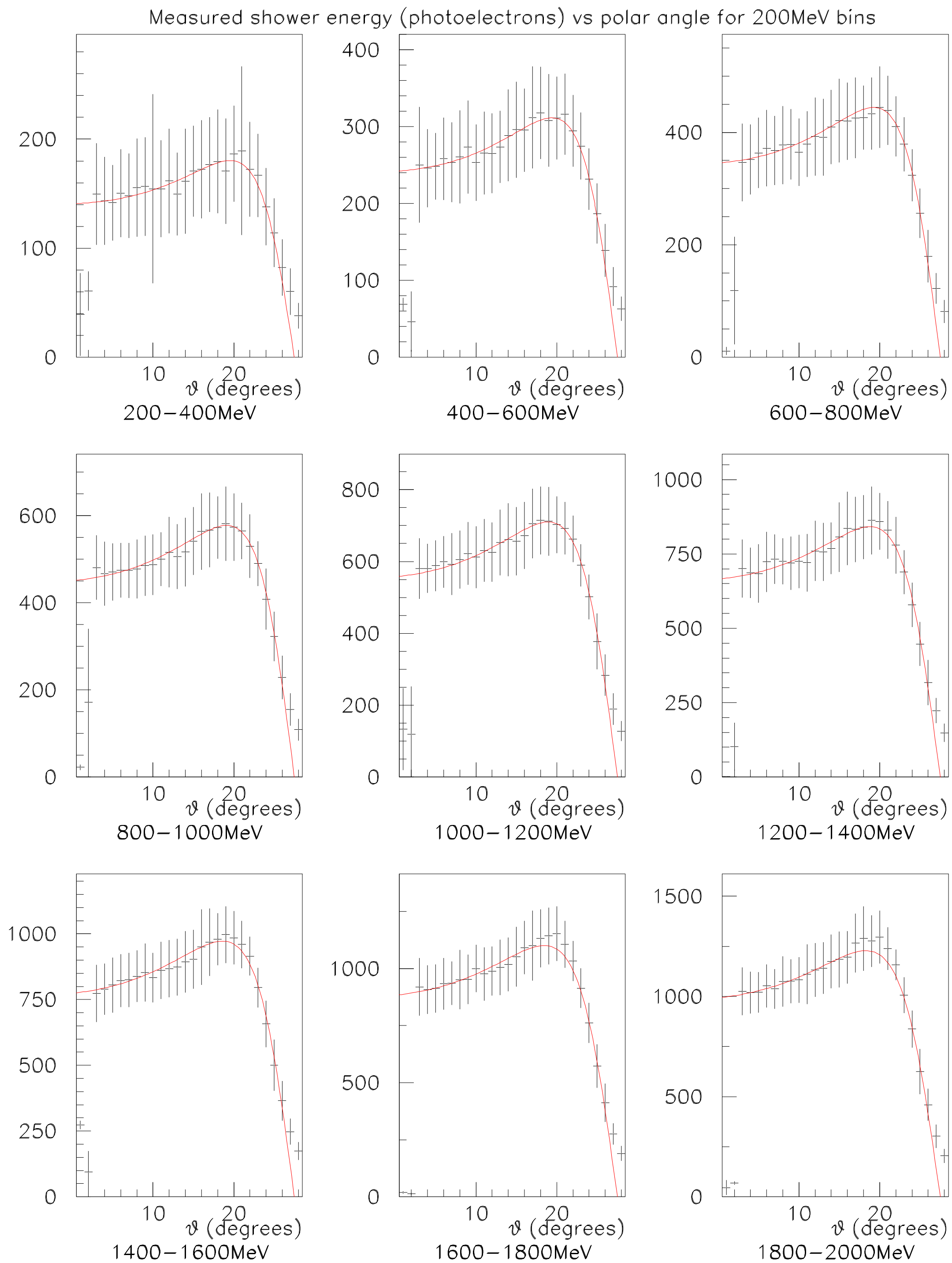

Figure 1: Total photoelectron yield of showers in the LGD, as a function of polar angle θ, for the given slices in E. The curves are the 2d-fit described in the text.

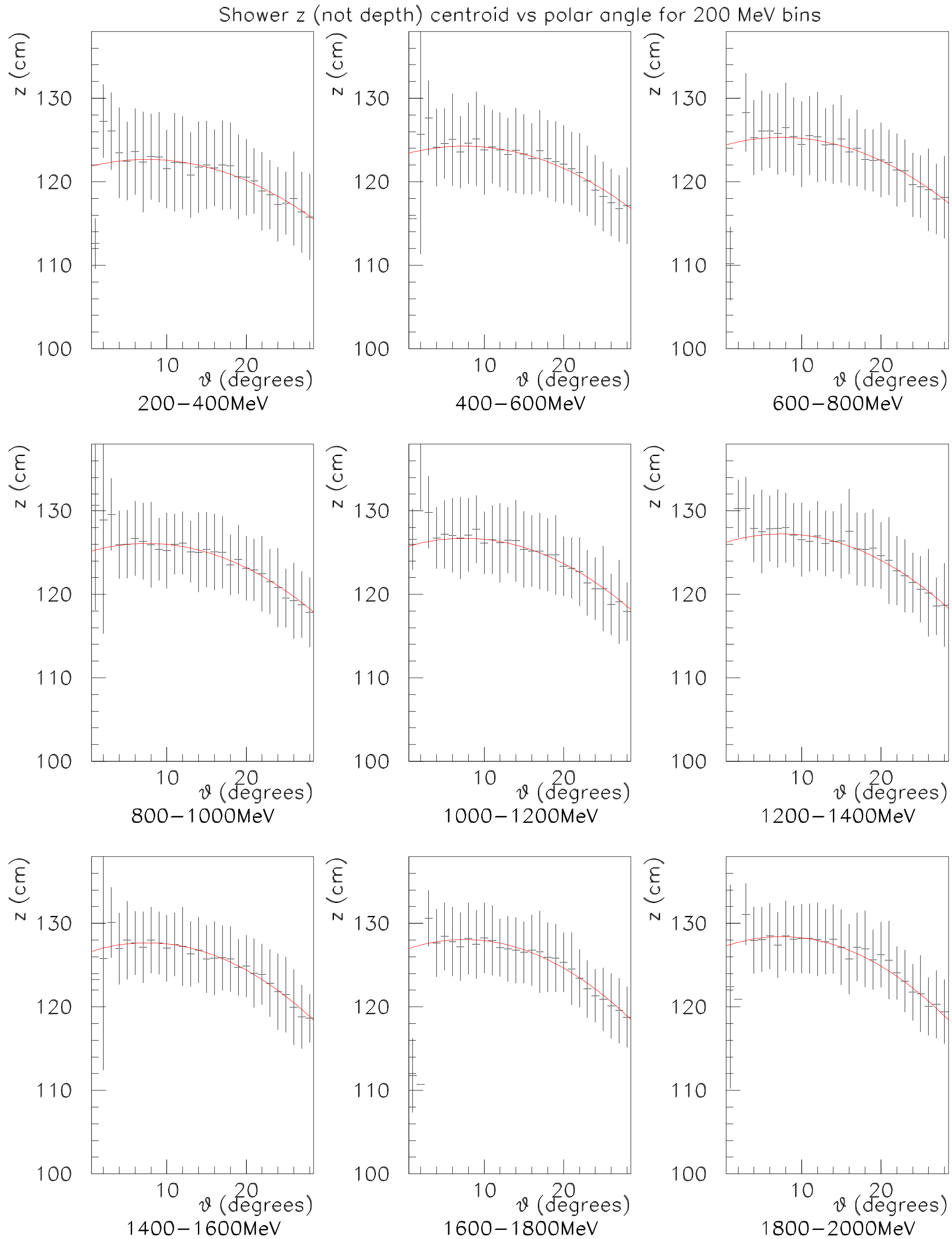

Figure 2: Third coordinate of the shower centroid inside the LGD, as a function of polar angle θ, for the given slices in E. Only light detected in the phototubes is counted in calculating the mean. Note that z is the coordinate along the beam direction, not the depth of the shower penetration along the shower axis.

where L is measured in photoelectrons and the angles θ, θ_0 and δ are in degrees. The value of θ_0 is the zero of the function, supposed to be the polar angle of the detector edge, and δ describes the width of the falling gain curve at the acceptance edge.

Values for each of the parameters θ_0, a, b, and δ were obtained for the 9 energy bins shown in Figs. 1-2, and these in turn were fitted *vs* energy. The results are given in Eq. 7

$$\begin{aligned} \theta_0 &= 27.5^0 \\ \delta &= 6.95 + 0.66E \\ a &= 7.4E \\ b &= 8900E^{1.11} \end{aligned} \tag{7}$$

The curves in Fig. 1 are the results of the above parameterization. Overall it gives a good description of the data. The points below 3^0 have been excluded from the fit because these showers start inside the beam hole. The points above 26^0 have also been excluded, which is about where one might guess from these data that the LGD acceptance ends. The biggest departure of the curve from the data occurs for the two highest energies in Fig. 1, where the data show a sharper maximum near 20^0 than the curves do. One might invoke another parameter to patch up that region, but I consider the discrepancy to be negligible, especially in view of the small angular region and the fact that it is at larger angles where the photon spectrum is mostly soft.

The function $Z(\theta, E)$ was obtained by a similar procedure. The simulation data for shower mean z were histogrammed on a grid of 200MeV $\times$ 1^0 bins. The slices in constant E were fitted to a function of θ and then the parameters were fitted as a function of E, as follows.

$$Z(\theta) = z_0 - q(\theta - \theta_m)^2 \tag{8}$$

where z_0 is given by Eq. 5 and the other two parameters are described by the linear functions given in Eq. 9. Z is measured in cm.

$$\begin{aligned} \theta_m &= 7.5^0 \\ q &= 0.015 + 0.0040E \end{aligned} \tag{9}$$

This parameterization is shown by the curves in Fig. 2.

4 Reliability

Certainly not everything in this Monte Carlo model is an accurate description of the Radphi calorimeter. The spectral attenuation of lead glass depends on its history and is almost certainly different for inner and outer blocks. The wrapping probably has air gaps in some places and none in others, which means that neither the black paper nor the aluminum foil models are really exact. Thus some checks are needed that the simulation gets the overall detector response right in regions where we already know the answer, before we can rely on it for guidance in the large-angle region where the response is unknown.

In the forward region $\theta < 10^0$ we already know how the shower depth z and yield p_e should depend on photon energy E. Up to an overall shift, z should be proportional to the radiation length times $\log E$. The simulation data are shown in Fig. 3 where only showers at $7^0 \pm 2^0$ have been included. Superimposed on the data are two curves, Eq. 5 in blue and the 2d parameterization in red. The two curves are difficult to distinguish at the resolution of this figure. This just reflects the fact that the depth calculation under the new parameterization is just the same as Eq. 5 in the forward region. Fig. 2 shows that $Z(\theta, E)$ continues smoothly from this region to smaller values at higher angles, partly from the geometric factor $\cos\theta$ and partly due to shower leakage.

In the forward region we also already know that the curve p_e vs E should look like Eq. 1. The simulation data are shown in Fig. 4 where only showers at $7^0 \pm 2^0$ have been included. Superimposed on the data are two curves, a free fit to Eq. 1 in blue and the 2d parameterization in red. Note that the free fit gives a value of $\epsilon = 0.055$, to be compared with a value of 0.06 that was obtained by D. Armstrong directly from Radphi data. The agreement between the red and the blue curves shows that the two embody essentially the same nonlinearity correction. Note that this was not put into the simulation by hand. It emerged from the interplay of internal reflection and attenuation effects on the one hand, and leakage effects on the other. At large angles one would expect leakage effects to win out over attenuation and cause the sign of ϵ to reverse. That is exactly what the simulation shows. In Fig. 5 is shown the same thing as Fig. 4 except that showers

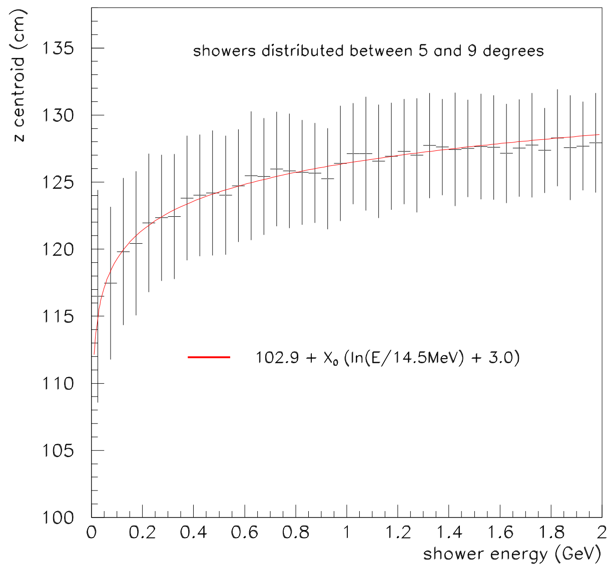

Figure 3: Same as Fig. 2, but plotted *vs* shower energy for fixed polar angle, instead of *vice versa*. The blue curve is the function in Eq. 5 and the red curve is the 2d parameterization described in the text. The two curves cannot be distinguished at the resolution of this plot.

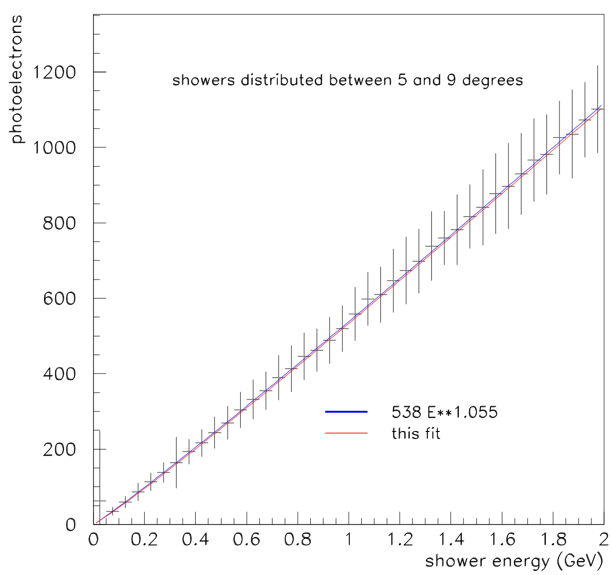

Figure 4: Same as Fig. 1, but plotted *vs* shower energy for fixed polar angle, instead of *vice versa*. The blue curve is the function in Eq. 1 with $\epsilon = 0.055$ and the red curve is the 2d parameterization described in the text.

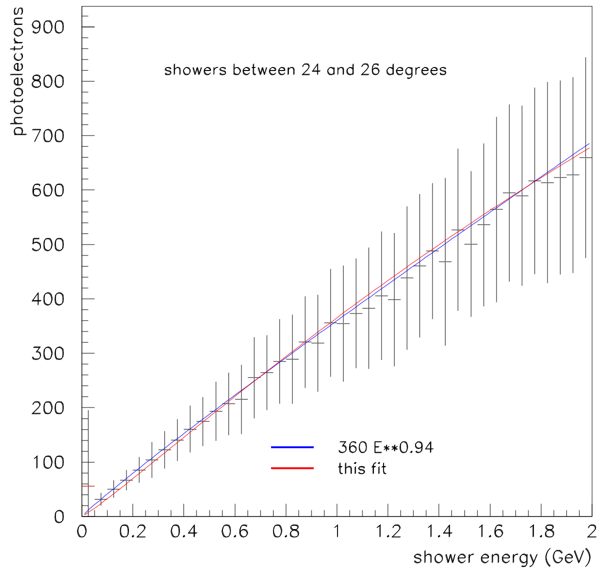

Figure 5: Same as Fig. 4, but for showers at high polar angle $\theta \approx 25^0$ where significant leakage is present. The blue curve is the function in Eq. 1 with $\epsilon = -0.062$ and the red curve is the 2d parameterization described in the text.

are selected in the angular range $25^0 \pm 1^0$. Here the free power-law fit gives an exponent less than 1. Again good agreement is obtained between the fit to Eq. 1 and the 2d parameterization.

In conclusion, the new parameterization will reproduce the present known corrections to shower depth and energy for showers at low polar angle. In addition, it will give us reliable guidance for obtaining corrected values of these variables for large-angle showers. In the remaining section of this report a procedure is presented for how Eq. 2-4 may be solved in analysis code.

5 New procedures

The code for solving Eqs. 2-4 is shown in Table 1. This code has been checked on real data and does not hang or crash on the first 10^6 events. The values for the energy and angle it returns look reasonable for the most part.

There is one case that is problematic, that being a high energy cluster whose center is at the outer edge of the detector. An example would be a single block on the outside edge of the detector with a pulse height corresponding to 2GeV. This is fairly rare, only a few per run, but they do have to be treated specially because in this case a solution to Eqs. 2-4 does not actually exist; no photon coming from the target, no matter the energy, would be able to deposit a mean of 2GeV in an outer block without scraping a block at a smaller radius. In real events, of course, not all photons come from the target, but this analysis assumes that they do. The result is that the iterative solution pushes the corrected energy up to ridiculous levels in search of a solution. The code in Table 1 fixes this by quitting out of the loop as soon as the solution energy exceeds some upper bound `MAX_PHOTON_ENERGY`.

Another feature of the code in Table 1 that was not presented in the discussion so far is the azimuthal dependence of the maximum polar angle θ_0 of the detector. In Eq. 7 the detector was assumed to be cylindrical. The code uses a simple quadratic fit to take into account the irregularities of the outer boundary of the actual LGD.

The code is located in routine `makePohotonPatch.c` and assumes that the cluster center (x, y) has already been stored in the structure `photon_VC` and that the summed cluster energy (uncorrected) is found in `cluster->energy`. Upon exit from the loop, the answers are stored in the variables `energy` and `theta`.

The new code has been uploaded to the central cvs repository. It replaces the former depth correction in

Table 1: Listing of c code for solving Eqs. 2-4.

```
#define TARGET_TO_LGD_DISTANCE 102.9
#define LGD_RADIATION_LENGTH 3.1
#define LGD_SHOWER_OFFSET 3.0
#define E_CRITICAL 0.01455
#define PE_PERGEV_LGD 540
#define MAX_PHOTON_ENERGY 5.6
#define RAD_DEG 57.29578
. . .
  float radius,pe,pe_seen,theta,energy,zed;
  float theta0,phi0,a,b,delta,z0,q,thetam;
  int niter = 0;
. . .
  energy = cluster->energy;
  zed = TARGET_TO_LGD_DISTANCE +
        LGD_RADIATION_LENGTH*(LGD_SHOWER_OFFSET + log(energy/E_CRITICAL));
  radius = sqrt(SQR(photon_VC.x)+SQR(photon_VC.y));
  theta = atan2(radius,zed) * RAD_DEG;
  pe_seen = energy*PE_PERGEV_LGD;
  pe = 0;

  /* find the edge of the detector at this azimuth */

  theta0 = 27.5;
  phi0 = atan2(photon_VC.y,photon_VC.x)*RAD_DEG + 180;
  phi0 -= 90*(int)((phi0+45)/90);
  phi0 = fabs(phi0);
  if (phi0 < 20) {
    theta0 += 0.38e-2*SQR(phi0);
  }
  else {
    theta0 += 0.21e-2*SQR(phi0-45);
  }
  while (fabs(pe-pe_seen) > 1) {
a = 7.4*energy;
b = 8900*pow(energy,1.11);
delta = 6.95+0.66*energy;
    pe = (theta0-theta)*(a + b/(SQR(theta0-theta)+SQR(delta)));
    energy *= pe_seen/pe;
z0 = TARGET_TO_LGD_DISTANCE +
             LGD_RADIATION_LENGTH*(LGD_SHOWER_OFFSET + log(energy/E_CRITICAL));
thetam = 7.5;
q = 0.015 + 0.004*energy;
    zed = z0 - q*SQR(theta-thetam);
    theta = atan2(radius,zed) * RAD_DEG;
    if (energy > MAX_PHOTON_ENERGY) {
      break;
    else if (++niter > 1000) {
      fprintf(stderr,"makePhotonPatch - maximum iteration count exceeded,");
      fprintf(stderr," giving up!\n");
      break;
    }
  }
```

makePhotonPatch.c and the energy nonlinearity correction in make_photons.c. So as not to give anyone an unwelcome surprise, the new code is not selected by default. Instead, the two files mentioned above were checked into cvs with special defines

```
#define OLD_DEPTH_CORRECTION true
#define OLD_NONLINEAR_CORRECTION true
```

in the head of the file. Commenting out these two lines will enable the new code. Note that they must be commented out in *both* makePhotonPatch.c and make_photons.c to obtain the new behavior.

As before, the gain coefficients that are obtained using this correction procedure in the calibration will give rise to raw cluster energies that are approximately equal to the corrected ones. If all behaves as expected, once calibration constants have been obtained with this correction in place, an analysis using those gain coefficients and this procedure will need no other patching up to obtain consistent results for the masses of known mesons in the Radphi data set.

References

[1] "Particle Physics Booklet", Particle Data Group (July 2000) 200.

[2] "Corrections to the Radphi mass scale", S. Teige, Radphi TechNote *1999-1201* (Dec 1999).

[3] "Estimation of the average depth of showers in the LGD", R.T. Jones, Radphi TechNote *2001-101* (Jan 2001).

[4] "Radphi LGD Calibration Logbook", R.T. Jones, J. Gunter, D. Armstrong, http://zeus.phys.uconn.edu/radphi/calibration/lgd.shtml (2001).