Technical Note
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Estimation of the average depth of showers in the LGD

Richard Jones
University of Connecticut, Storrs CT

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Abstract

The polar angle of a photon from the Radphi target producing a shower in the lead-glass detector (LGD) depends upon all three coordinates (x, y, z) of the shower centroid in the LGD. In the coordinate system with its origin in the center of the target and its z-axis pointing along the incident beam direction, only x and y are directly determined from the data while z has to be inferred from the shower energy. In this note a combination of shower simulations and phenomenology is used to motivate a recommended way of calculating the z of a shower. This result is compared with the method that is found in the code that has been in use up to the time this note was written.

1 Shower phenomenology

The distribution of energy deposition in electromagnetic showers generated by high-energy electrons and photons is summarized by the Particle Data Group [1]. In their review is given the following expression for the shower profile as a function of penetration depth t into a material.

$$-\frac{dE}{dt} = E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)} \tag{1}$$

Here b is a constant that depends on the material, but is approximately $(2X_0)^{-1}$ where X_0 is the radiation length of the material. The variable a in Eq. 1 depends on the energy of the shower through the relation

$$t_{max} = \frac{a-1}{b} = X_0 \left[\ln\left(\frac{E_0}{E_c}\right) + C_j \right] \tag{2}$$

where E_0 is the incident gamma energy, E_c is the critical energy of the shower medium, and the constant C_j is +0.5 for incident photons and -0.5 for electrons or positrons. The distribution given by Eq. 1 measures the energy deposition via ionization by all shower particles at a given depth, which is not in general identical to the Čerenkov light emission rate. However electrons remain relativistic for most of the length of their track, in which case both the ionization energy loss and the Čerenkov emission rate are proportional to the total $e^{\pm}$ track length.

The quantity of interest in reconstructing the polar angle from cluster measurements in the LGD is the average depth of the Čerenkov emission in the LGD. This is not exactly the quantity t_{max} in Eq. 2 but the two are closely related. From Eq. 1 one can see that the quantity t_{max} in Eq. 2 is just the depth at which the energy deposition rate reaches its maximum. Using Eq. 2 to define a allows us to go back to Eq. 1 and calculate the average deposition depth t_{ave} which one finds is simply a/b. This leads to a formula that gives t_{ave} for gamma showers in terms of the shower energy E_0 and physical constants.

$$t_{ave} = X_0 \left[\ln\left(\frac{E_0}{E_c}\right) + 0.5 + \frac{1}{b} \right] \tag{3}$$

2 Simulations

The phenomenology described above contains a few assumptions that can be checked with a detailed Monte Carlo simulation. For this purpose I adapted the *Gradphi* program to write out detailed information regarding the tracking within a single shower in the LGD. GEANT was instructed to track each electron down to 100keV of kinetic energy. Any bremsstrahlung photons over 100keV generated during tracking were individually simulated. A shower from a 1GeV gamma ray at normal incidence contained about 160 separate $e^{\pm}$ tracks in the glass. Each track was broken up into segments of length about 1mm and entered *vs* depth into a histogram weighted by track segment length. This histogram represents the Čerenkov emission rate as a function of depth. The result for 10000 1GeV showers at normal incidence is shown in Fig. 1.

Fig. 1 shows the average longitudinal profile of the showers but does not indicate the scale of the fluctuations from one shower to the next. In Fig. 2 is shown the mean depth from Fig. 1 calculated individually for each event. As expected, the mean is in the same place and the standard deviation has decreased. Note that the standard deviation does not decrease as $1/\sqrt{160}$ because the tracks are not independent. Most of the fluctuations are tied to the variations in the conversion depth of the primary gamma ray.

The fit in Fig. 1 is of the form Eq. 1 with b allowed to vary. The best value for b of 0.54 is in good agreement with expectations [1]. The fit was done with 2 parameters, b and a vertical scale factor. It is clear from the figure that a significant amount of shower energy escapes out the back of the block. This fact was known by the I.U. team that designed the calorimeter. The choice of 15 radiation lengths was made to compromise between shower containment and attenuation of Čerenkov light in the glass. In Radphi the shower leakage gets somewhat better as the polar angle of the shower increased from normal incidence up to about 20^o beyond which it abruptly increases due to leakage out the sides. Correcting the shower energy for leakage is an important matter for a future study. Only showers up to 20^o will be considered in this study.

3 Existing treatment

The Radphi analysis code that calculates the penetration depth of the showers in the LGD is found in the file `makehits/makePhotonPatch.c`. The relevant part of the code, as it exists now, is shown below.

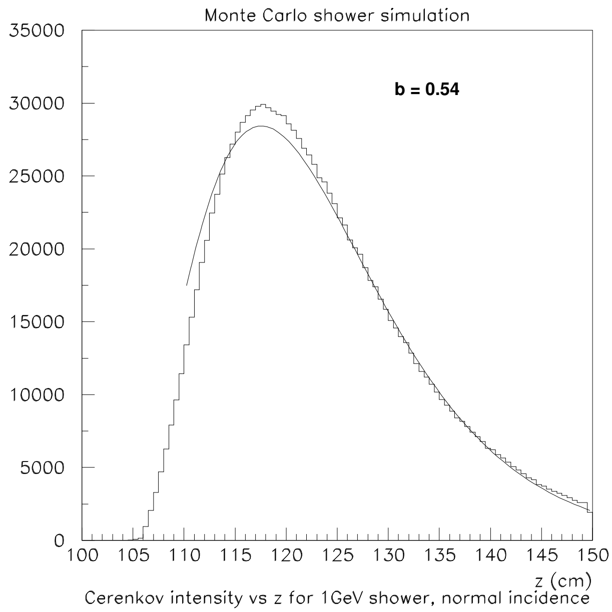

Figure 1: Simulated Čerenkov emission intensity em vs penetration depth for 1.0GeV gamma-ray showers at normal incidence. The front face of the LGD is located at 102.9cm. The simulation contains 10000 showers.

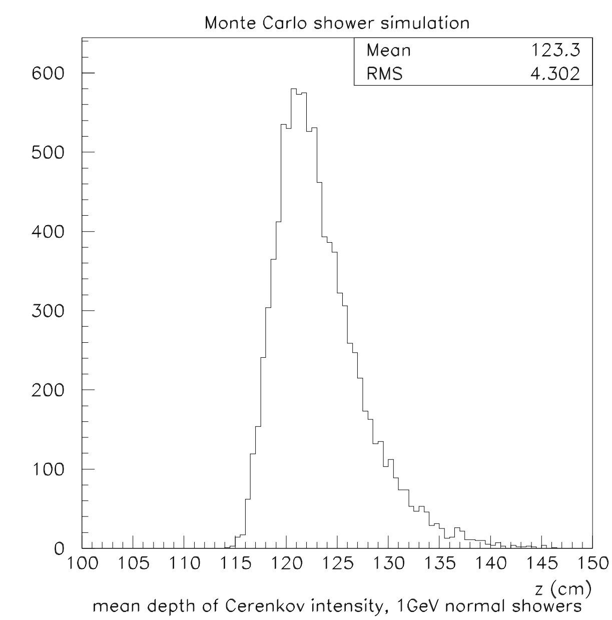

Figure 2: Mean depth of the Čerenkov emission profile, calculated separately for each event and then histogrammed, for the 10000 showers shown in Fig. 1. The fluctuations are mainly determined by the conversion depth of the primary gamma ray.

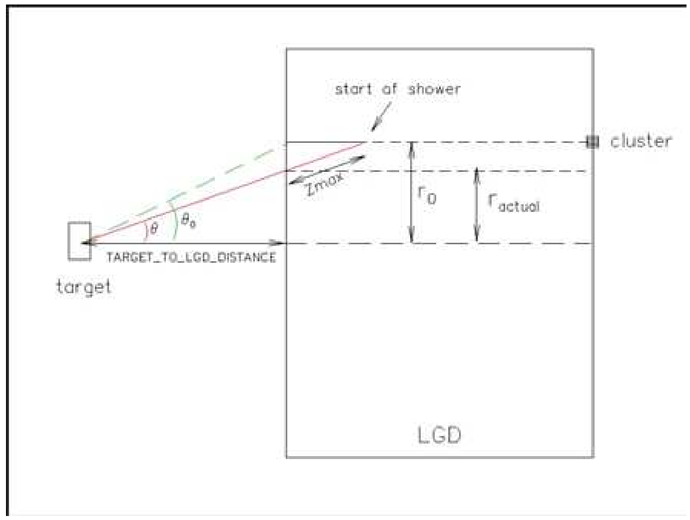

Figure 3: Various forms for the relation between the z of the shower centroid and the shower energy, plotted for showers incident at 20^o from the normal: the phenomenological formula recommended in this note (red), a simplified approximation to the former (black), the treatment implemented in the present code (blue), and the present treatment with the corrected target-LGD distance (green). *Figure courtesy of D. Armstrong*

```
------------------------------
#define TARGET_TO_LGD_DISTANCE 108.0
#define ZMAX_MULT 1.06
#define ZMAX_COEFF 4.52
#define E_CRITICAL 0.01455
...
  zMax = ZMAX_MULT*ZMAX_COEFF*log(cluster->energy/E_CRITICAL);
------------------------------
```

The meaning of the parameter `zMax` is shown in Fig. 3. Further down in the code it is used to project the measured (x, y) of the cluster onto the plane defined by z=`TARGET_TO_LGD_DISTANCE` which nominally corresponds to the front face of the glass. The calculated value of `zMax` may be used to find the z coordinate (beware the notation) of the shower centroid according to Fig. 3. This is shown in Fig. 4. The blue curve represents the code listed above. The data point and error bar represent the Monte Carlo distribution from Fig. 2 rotated to 20^o. The blue curve misses the data because the value of `TARGET_TO_LGD_DISTANCE` is too large. Survey measurements give 102.9cm for this parameter. Just changing that parameter gives the green curve. The red curve is just Eq. 3 applied to an incidence angle of 20^o.

4 Recommendations

Based on this study, I would recommend that the code in `makePhotonPatch.c` be changed as follows to agree with Eq. 3.

```
------------------------------
#define TARGET_TO_LGD_DISTANCE 102.9
#define LGD_RADIATION_LENGTH 3.1
#define LGD_SHOWER_OFFSET 2.35
#define E_CRITICAL 0.01455
...
  zMax = LGD_RADIATION_LENGTH*(LGD_SHOWER_OFFSET + log(cluster->energy/E_CRITICAL));
------------------------------
```

As a check, I applied this change to the code and then generated a sample of 10000 showers from gamma rays at normal incidence and random energies between 0 and 2GeV. For each shower I compare the actual

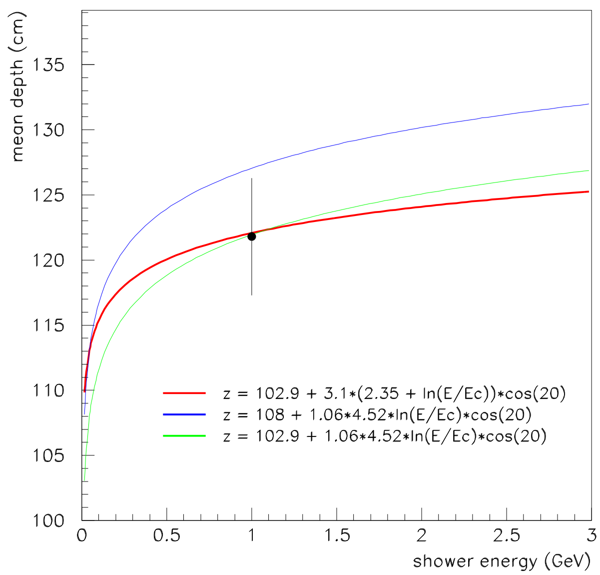

Figure 4: Various forms for the relation between the z of the shower centroid and the shower energy, plotted for showers incident at 20^o from the normal: the phenomenological formula recommended in this note (red), a simplified approximation to the former (black), the treatment implemented in the present code (blue), and the present treatment with the corrected target-LGD distance (green).

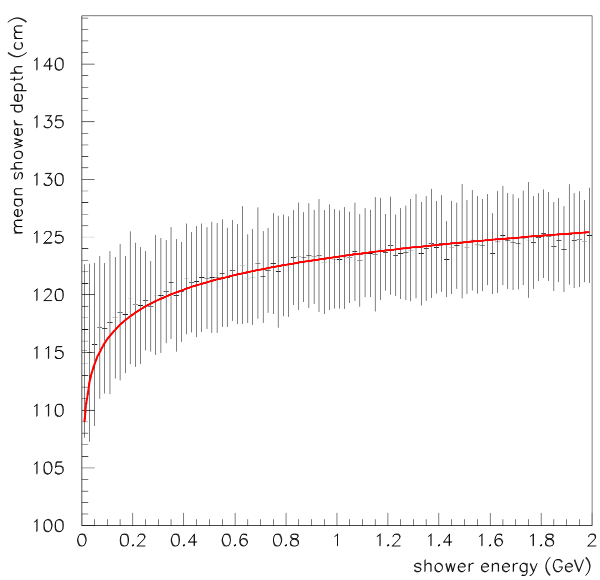

Figure 5: Comparison between the shower centroid from the Monte Carlo generator (data points with error bars indicating r.m.s. fluctuations) with that calculated by the recommended `makePhotonPatch` code.

depth centroid returned by the generator with the value of `zMax` computed by `makePhotonPatch`. The comparison is shown in Fig. 5.

References

[1] "Particle Physics Booklet", Particle Data Group (July 2000) 200.