Technical Note
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Kinematic fit to diffractive hypotheses for Radphi

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Abstract

A kinematic fit accomplishes two things. First it provides a statistical test of the agreement between a measured final state and a kinematic hypothesis. Second it produces a complete reconstruction of the final state consistent with the hypothesis, including values for unmeasured variables and modified values for the measured variables which represent an optimal compromise between the measurements and the kinematic constraints implied by the hypothesis. Of particular interest to Radphi are the class of diffractive reactions written as $\gamma p \rightarrow pX$ where $X \rightarrow n\gamma$. This note outlines the parameters and constraints that implement a kinematic fit of Radphi events to this type of hypothesis.

The problem at hand is simplified by considering separately the quantities that describe the final state from the quantities that are measured or implied from kinematics. In this text the former are called *parameters* because they will be the quantities to be varied during optimization. Contrasted to the parameters are the quantities that comprise the terms in the penalty function, which are described below under the headings of *measured variables* and *kinematic constraints.*

1 Kinematic parameters

The most direct way to describe the final state of $(p, n\gamma)$ is as $(n+1)$ 3-vectors which are written in the lab frame as $\vec{p}$, $\vec{k}_1$, $\vec{k}_2, ..., \vec{k}_n$. The corresponding kinetic energies are written in terms of these parameters as $\sqrt{p^2+m_N^2} - m_N$, k_1, $k_2, ..., k_n$. A complete description would also require another momentum vector $\vec{p}_R$ to represent the final-state momentum carried off by the recoil $A=8$ system. An alternative to including $\vec{p}_R$ in the final state is to consider only the γ, p system in the final state, and treat $-\vec{p}_R$ as the momentum of the proton in the initial state. This turns out to be convenient because the Radphi resolution is not sufficient to resolve $\vec{p}_R$ and it plays the role of a kind of uncertainty on the initial state momemtum. How this works out is shown in section 3 below. The description is complete when the incident photon in include with its direction fixed along the z-axis and its energy represented by parameter k_0. In all, this gives $4+3n$ parameters.

2 Measured variables

For each reconstructed cluster in the forward calorimeter state one measures more or less independently the direction and energy of the shower. For the purposes of this discussion, measured variables are represented by capital letters. The direction of a shower is most simply expressed by the direction cosines, of which the x and y components are independent and the z is implied. Therefore the measured gamma variables are listed individually as $K_1, ..., K_n$, $X_1, ..., X_n$ and $Y_1, ..., Y_n$, where $X_i = \hat{K}_i \cdot \hat{\imath}$ and $Y_i = \hat{K}_i \cdot \hat{\jmath}$. The recoil proton direction is measured in polar coordinates rather than Cartesian, so its variables are designated Θ_p and Φ_p. To a good approximation the measurement errors on these variables are independent and are represented below with the Δ prefix, as in ΔK_1 or $\Delta\Theta_p$. In the initial state, the energy of the tagged photon is designated K_0. In all, this gives $3+3n$ measurements.

3 Kinematic constraints

Constraints come from two sources: energy-momentum conservation from the initial state and mass constraints amoung the $n\gamma$ coming from the final-state hypothesis. Since there is always as least one mass constraint of the latter type, this means that there are at least 4 more constraints than unmeasured parameters, so that a kinematic fit can provide a powerful filter to classify events. Like the measurements, it turns out that all of the constraints have some uncertainty associated with them. The energy-momentum constraints are as follows.

$$\vec{p}\cdot\hat{\imath} + \sum_{i=1}^{n} \vec{k}_i \cdot \hat{\imath} = 0 \pm 250\mathrm{MeV}/c \tag{1}$$

$$\vec{p}\cdot\hat{\jmath} + \sum_{i=1}^{n} \vec{k}_i \cdot \hat{\jmath} = 0 \pm 250\mathrm{MeV}/c \tag{2}$$

$$\vec{p}\cdot\hat{k} + \sum_{i=1}^{n} \vec{k}_i \cdot \hat{k} = k_0 \pm 250\mathrm{MeV}/c \tag{3}$$

$$\sqrt{p^2+m_N^2} + \sum_{i=1}^{n} k_i = k_0 - 10\ \mathrm{MeV} \pm 10\mathrm{MeV} \tag{4}$$

The ± 250 MeV/c uncertainties come from the Fermi motion of the initial proton in the nuclear target. The -10 ± 10 MeV offset in the energy conservation constraint accounts for the nuclear binding of the initial proton. For each decaying meson in the final state hypothesis there is an additional constraint of the following general form.

$$\sum_{i}^{M_m} k_i^2 - \left| \sum_{i=1}^{M} \vec{k}_i \right|^2 = \mu_m^2 \pm \mu_m \Gamma_m \tag{5}$$

This represents a meson m of central mass μ_m and width Γ_m which decays through a chain that leads finally to M_m final gammas.

4 Penalty function

The following function represents the deviation of the parameters from the measurements and kinematic constraints.

$$\begin{aligned} f \quad = \quad & \sum_{i=1}^{n} \left(\frac{K_i - k_i}{\Delta K_i} \right)^2 + \sum_{i=1}^{n} \left(\frac{X_i - \hat{k}_i \cdot \hat{\imath}}{\Delta X_i} \right)^2 + \sum_{i=1}^{n} \left(\frac{Y_i - \hat{k}_i \cdot \hat{\imath}}{\Delta Y_i} \right)^2 \\ & + \left(\frac{\Phi_N - \tan^{-1} \frac{\vec{p} \cdot \hat{\jmath}}{\vec{p} \cdot \hat{\imath}}}{\Delta \Phi_N} \right)^2 + \left(\frac{\Theta_N - \tan^{-1} \frac{\sqrt{(\vec{p} \cdot \hat{\imath})^2 + (\vec{p} \cdot \hat{\jmath})^2}}{\vec{p} \cdot \hat{k}}}{\Delta \Theta_N} \right)^2 \\ & + \left(\frac{1}{250 \text{ MeV}/c} \right)^2 \left[\left(\vec{p} + \sum_{i=1}^{n} \vec{k}_i \right) \cdot \hat{\imath} \right]^2 \\ & + \left(\frac{1}{250 \text{ MeV}/c} \right)^2 \left[\left(\vec{p} + \sum_{i=1}^{n} \vec{k}_i \right) \cdot \hat{\jmath} \right]^2 \\ & + \left(\frac{1}{250 \text{ MeV}/c} \right)^2 \left[\left(\vec{p} + \sum_{i=1}^{n} \vec{k}_i \right) \cdot \hat{k} - k_0 \right]^2 \\ & + \left(\frac{1}{\Delta K_0} \right)^2 \left(K_0 - \sqrt{p^2 + m_N^2} - \sum_{i=1}^{n} k_i - 10 \text{ MeV} \right)^2 \\ & + \sum_{m=1} \left(\frac{\sum_i^{M_m} k_i^2 - \left| \sum_i^{M_m} \vec{k}_i \right|^2 - \mu_m^2}{\mu_m \Gamma_m} \right)^2 \end{aligned} \tag{6}$$