Towards constrained fitting for RadPhi
April 27, 2000

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Abstract

Efforts to determine quanties useful for constraints for Radϕ are summarized.

1 Introduction

In addition to the measurement of photon energies and momenta other physical quantities are measured by our apparatus. For example, the direction of the recoil proton is determined by the location of the target and the location of a pixel in the BSD. The beam energy is determined by the index of the struck paddle in the tagger. In this note I summarize progress in determining the parameters that turn detector indicies into useful physical quantities.

2 The tagger

Figure 1 show how the photon beam energy can be determined by the tagger. To a very good approximation the photon energy is given by

$$E_\gamma = (0.9481 - 0.0094 \times I_t) \times E_{electron} \tag{1}$$

where I_t is the index (0-18) of the t-counter. The values used in this fit are given in table 1.

3 The BSD

The direction of the recoil proton is given by a unit vector from the center of the target to the center of the pixel in the BSD struck by the proton. A pixel is determined by a triple coincidence of a "straight", "left" and "right" paddles. Software to find such coincidences exists.

The azimuthal angle of a pixel is determined by by the straight paddle in the coincidence. This angle in the lab can be determined demanding conservation of momentum. A set of events with well reconstructed $\omega \to \pi^0\gamma$ decays

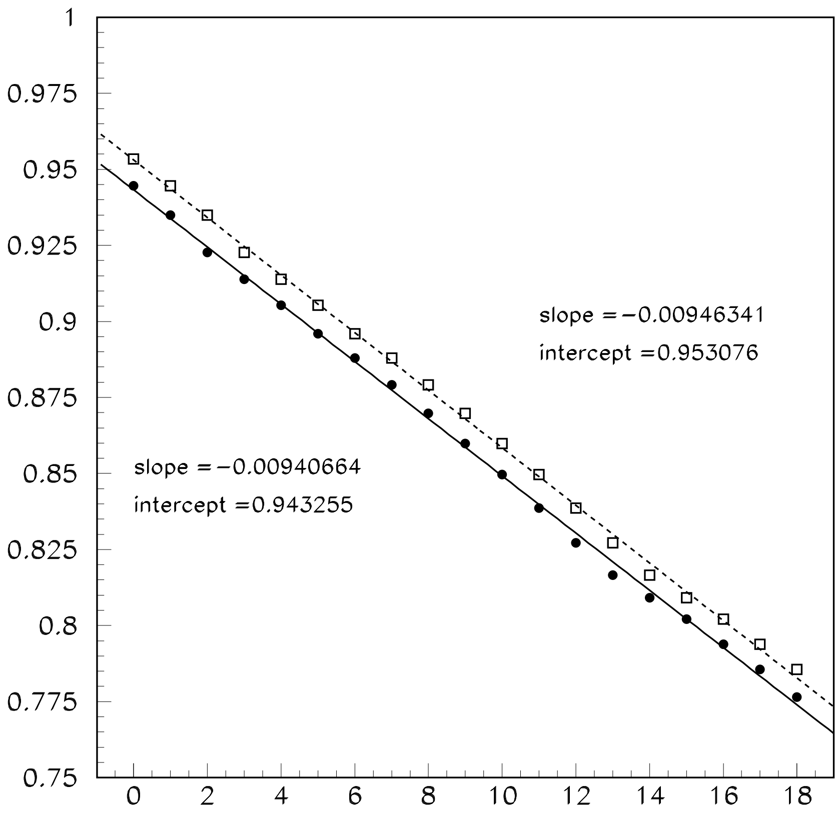

Figure 1: Fits to the tagger t-counter energy fraction limits as a function of t-counter index. The vertical axis is the fraction of the electron beam energy taken by a beam photon. The horizontal axis is the t-counter index index. The two lines give the upper and lower limits of the energy fraction subtended by the counters.

T-counter Index	Lower limit	Upper limit
0	.9446	.9534
1	.9349	.9446
2	.9227	.9349
3	.9139	.9227
4	.9053	.9139
5	.8959	.9053
6	.8880	.8959
7	.8792	.8880
8	.8697	.8792
9	.8599	.8697
10	.8496	.8599
11	.8386	.8496
12	.8272	.8386
13	.8166	.8272
14	.8092	.8166
15	.8021	.8092
16	.7938	.8021
17	.7856	.7938
18	.7765	.7856

Table 1: The upper and lower energy fractions of the t-counters as a function of t-counter index. Data provided by Dan Sober.

was selected. It was further required that the events had exactly one pixel. The azimuthal angle of the reconstructed ω momentum was plotted under the condition a certain straight paddle was struck. The mean value of the ω azimuthal angle was plotted as a function of the index of the struck straight paddle. Figure 2 shows the result of this fit.

The expected azimuthal angle of the momentum a contained event with a single pixel is given by

$$\phi_F = -2.7217 + 0.25904 \times I_S \tag{2}$$

where I_S is the index (0-23) of the struck straight paddle. Conservation of momentum (and neglecting the target Fermi momentum) implies the azimuthal angle of the recoil proton is given by

$$\phi_p = -2.7217 + 0.25904 \times I_S + \pi. \tag{3}$$

The polar angle of the pixel is determined by the right and left paddles in the pixel. The pixel reconstruction software returns an index, I_z, between 0 and 8 giving the number of the pixel ring struck. Geometric arguments translate this index into a z-position in the lab

$$Z_p = F(I_z) \times B_l + z_o \tag{4}$$

where z_o is an offset of the upstream end of the BSD relative to the target, B_l is the length of the BSD and $F(I_z)$ is given by

$$\begin{array}{rc} 1/16 & I_z = 0 \\ I_z/8 & 1 \leq I_z \leq 7 \\ 15/16 & I_z = 8. \end{array} \tag{5}$$

Measurement gives $z_o = 0$ and $B_l = 70\ cm$. The polar angle of the pixel can be calculated from Z_p and the radius of the BSD, R_{BSD}, (taken to be 36 cm) since

$$\tan\theta_p = Z_p / R_{BSD}. \tag{6}$$

4 The electron beam energy

Equation 1 contains the electron beam energy. This should be known but it can also be determined independently from the data. Conservation of momentum requires

$$\vec{B} + \vec{T} = \vec{F} + \vec{R} \tag{7}$$

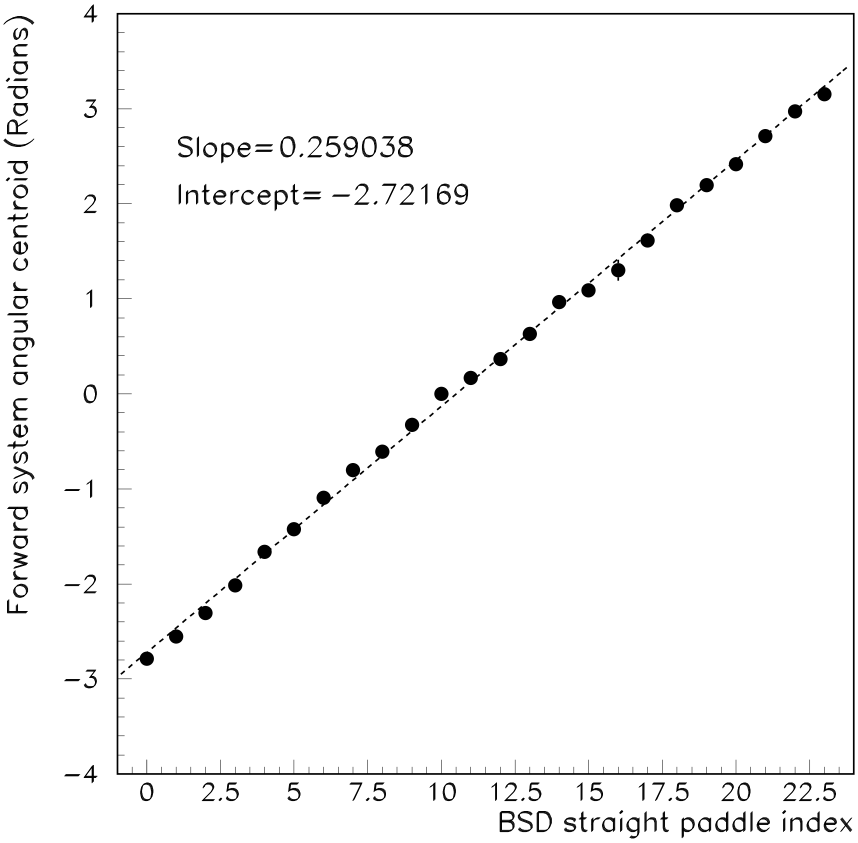

Figure 2: The mean value of the azimuthal angle of well contained $\omega \to \pi^0\gamma$ events as a function of the straight paddle index in the BSD.

where $\vec{B}$ is the beam momentum, $\vec{T}$ is the momentum of the target, $\vec{F}$ is the momentum of the forward system, that is, the sum of all observed photon momenta, and $\vec{R}$ is the momentum of the recoil proton. All quantities in eqn. 7 are known except the magnitude of the proton recoil momentum. This quantity can be determined by minimizing the magnitude of the missing momentum.

Defining the missing momentum as

$$\vec{M} = \vec{F} + \vec{R} - \vec{B} - \vec{T} \tag{8}$$

the magnitude of the recoil momentum is determined by

$$\frac{\partial \mid \vec{M} \mid^2}{\partial p_r} = 0. \tag{9}$$

Eqn. 9 has solution

$$p_r = U_z E_\gamma - \vec{U} \cdot \vec{F} \tag{10}$$

where $\vec{U}$ is a unit vector from the target to the recoil proton pixel. The missing momentum can now be evaluated. E_γ is given by eqn. 1 which contains $E_{electron}$ which is treated as a free parameter. $\mid \vec{M} \mid$ is evaluated for many events and for several choices of $E_{electron}$. The result is shown in figure 3. Since $\mid \vec{M} \mid$ is zero for a perfectly contained, perfectly measured event, the value of E_γ that minimizes $\mid \vec{M} \mid$ is the best choice for this parameter.

Similar arguments apply to the components of $\vec{M}$. Figures 4 and 5 show the transverse and logitudinal components of $\vec{M}$ as a function of $E_{electron}$. The values for $E_{electron}$ found by these analyses agree reasonably well and give a value of 4.40 ± 0.03 GeV for the endpoint energy.

5 Discussion

The result for the beam endpoint energy is correlated with the energy scale of the LGD as can be seen from eqn. 10. The energy scale of the LGD is in turn determined by the observed masses of several narrow resonances observed in the data. Given a known endpoint energy the absolute energy scale of the LGD can be adjusted (within some limits) to reproduce the known endpoint energy by adjusting the target to LGD distance. Figures 6 and 7 show how this is possible. The different values for the slope obtained for the η and the ω limit the range of adjustment available for the LGD energy scale. Moving the nominal target to LGD distance (and rescaling the LDG energy) by too much

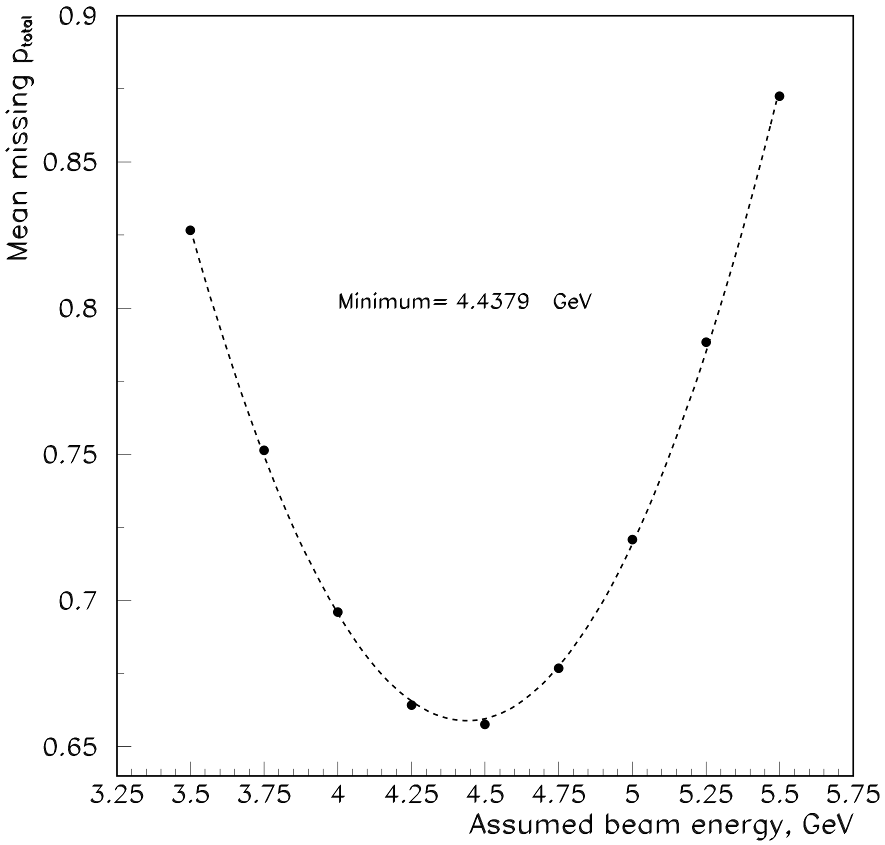

Figure 3: The mean value of $| \vec{M} |$ as a function of the presumed electron energy. The fit is to a second order polynomial.

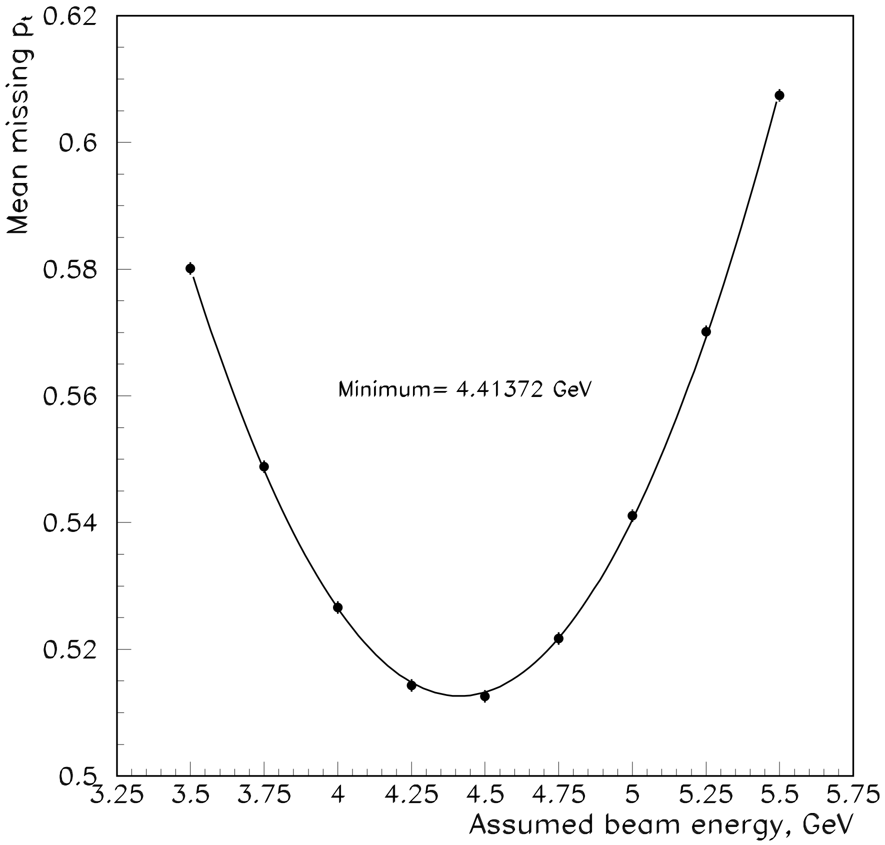

Figure 4: The mean value of M_t as a function of the presumed electron energy. The fit is to a second order polynomial.

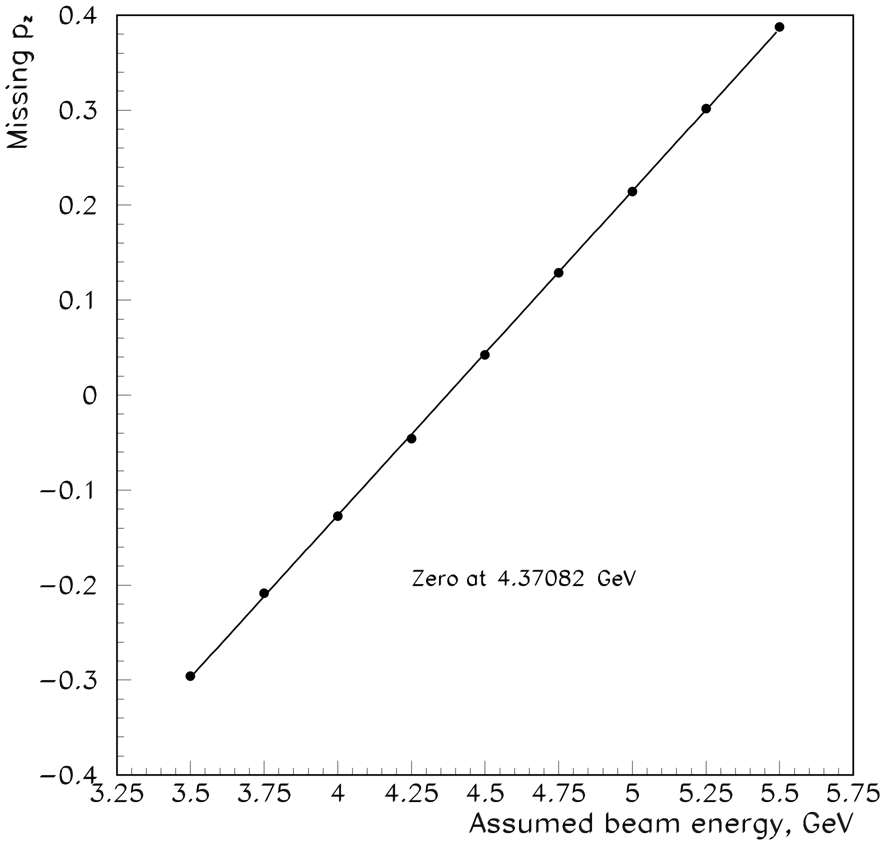

Figure 5: The mean value of M_z as a function of the presumed electron energy. The fit is to a first order polynomial. Unlike $\mid \vec{M} \mid$ and M_t which are strictly positive, M_z is a signed quantity that is zero for a perfectly measured, perfectly contained event.

causes a situation where both masses do not agree with the known values for these masses.

The target to LGD distance is a measured quantity but the initial depth of the shower in the LGD is a parameter of the depth correction. Neither are subject to large changes but a few centimeters might not be unreasonable. The derivitives of the mass scale shown in figures 6 and 7 show that an adjustment of, at most, a few tens of MeV in the endpoint energy are possible. These points, taken together, show the importance of an independent determination of the beam endpoint energy.

In other words, I think I've predicted the electron beam energy. We should check this out to see if I'm all wet or something is wrong with this analysis. So if anyone knows the answer, please tell me.

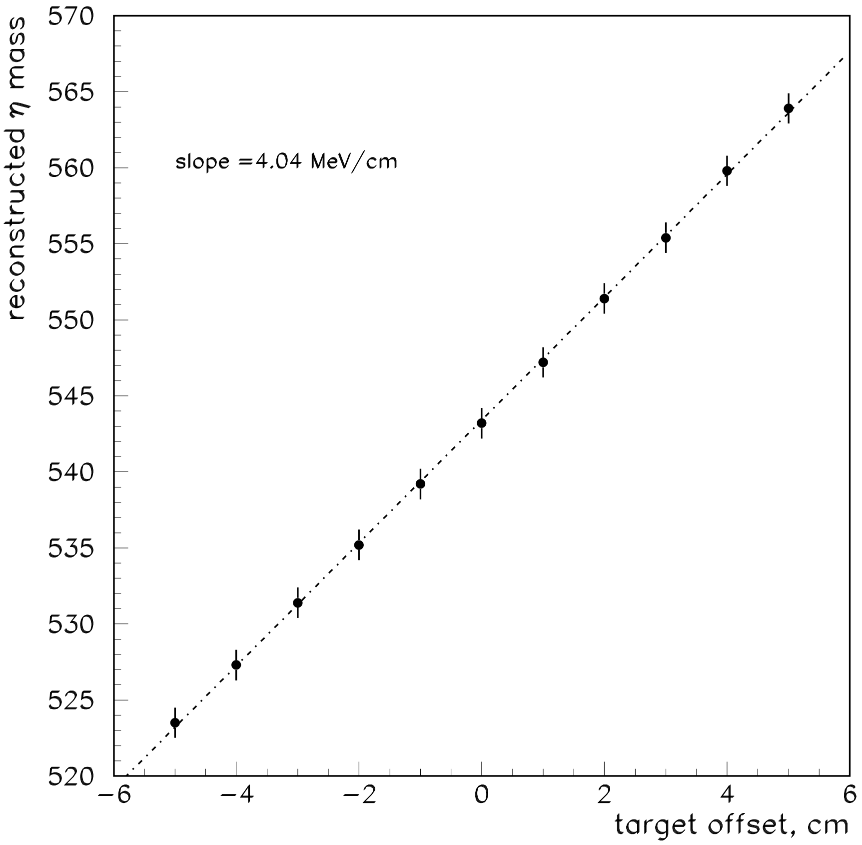

Figure 6: The effect of choosing a different target to LGD distance on the observed mass of the η.

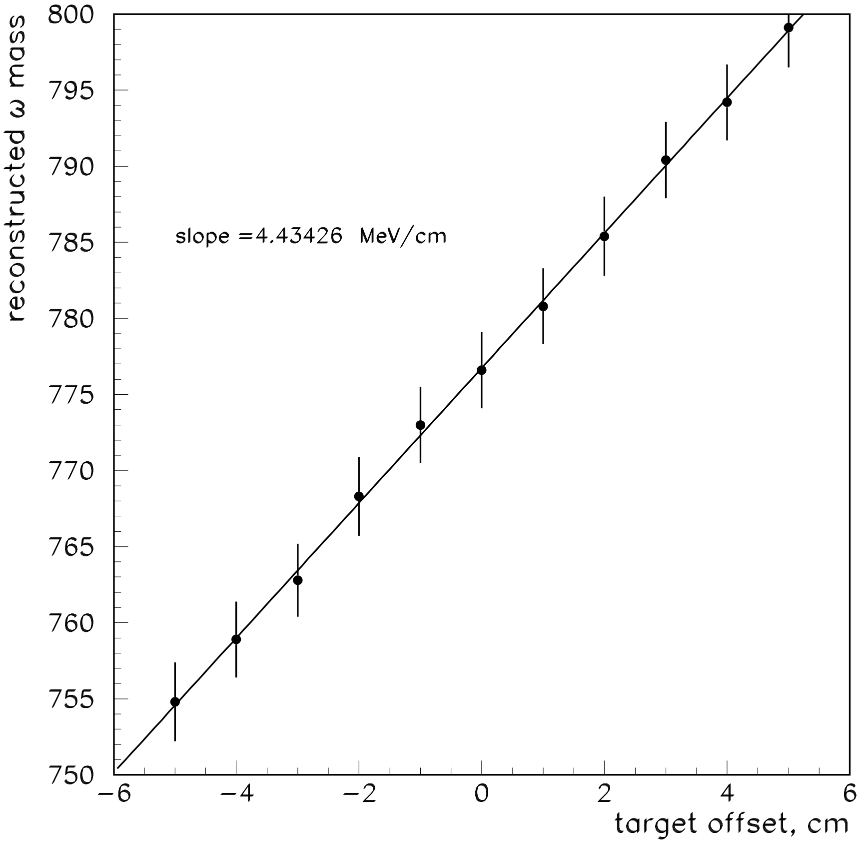

Figure 7: The effect of choosing a different target to LGD distance on the observed mass of the ω.