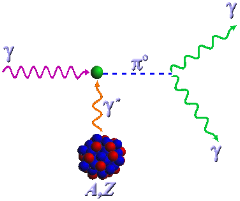
Chiral Anomalies and their rôle in the π^0, η and η' Mesons

Jose L. Goity
Hampton University/Jefferson Lab

PHP 2008 Workshop



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Outline

- Chiral symmetries and anomalies in QCD
- Interplay of Goldstone Bosons and anomalies
- The π^0, η and η' trio and their $\gamma\gamma$ decays
- Why more precision is useful
- Other processes
- Summary

Chiral Symmetries and Anomalies

QCD Lagrangian

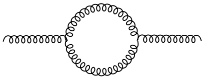
$$L_{QCD} = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + i\bar{q}\, \not{D}(G) q + \bar{q} \mathcal{M}_q\, q$$

Global Symmetries for $\mathcal{M}_q = 0$

Scale/Conformal$\times SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1)$

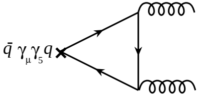
Anomalies

Scale Anomaly



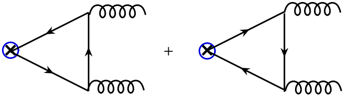
$$\Theta^\mu_\mu = \frac{\beta(g)}{2g}\, G^{\mu\nu} G_{\mu\nu}$$

Axial Anomaly



$$\partial^\mu A_\mu = \frac{N_f \alpha_s}{2\pi}\, G^{\mu\nu} \tilde{G}_{\mu\nu}$$

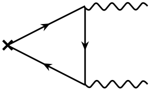
UV origin of anomalies



Both diagrams linearly UV divergent; cancellation up to finite term, identified with anomaly

EM Chiral Anomalies

Photon couplings to quarks generate anomalies for the three axial currents $\bar{q}\gamma_\mu\gamma_5\, q$, $\bar{q}\gamma_\mu\gamma_5\lambda_3\, q$, and $\bar{q}\gamma_\mu\gamma_5\lambda_8\, q$



$$\partial^\mu A^a_\mu = \frac{N_c\alpha}{2\pi}\; Tr(\hat{Q}^2\lambda_a)\, F^{\mu\nu}\tilde{F}_{\mu\nu}$$

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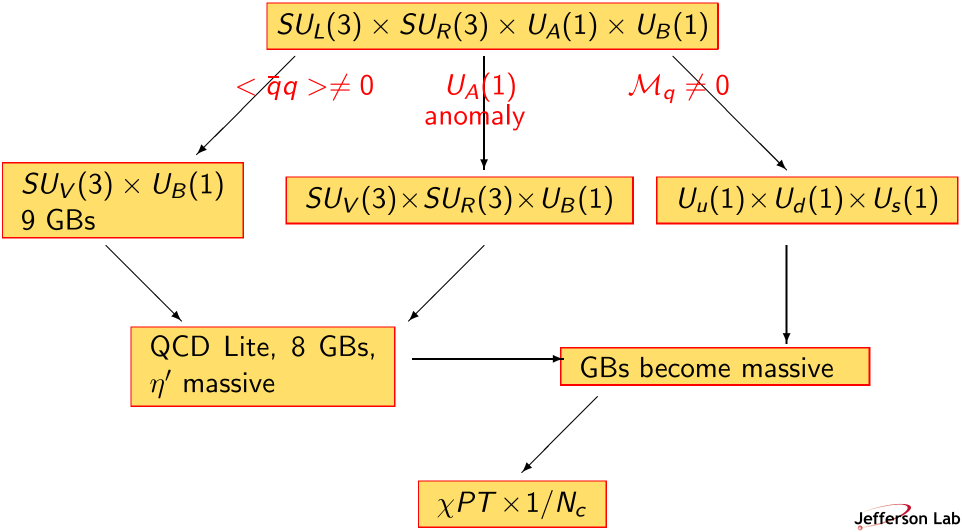
General properties of the axial and chiral anomalies

- They are preserved under loop corrections
- They must be the same at the quark-gluon level as at the hadronic level: anomaly matching
- Goldstone Bosons coupling to anomalous currents are the key to the matching of chiral anomalies
- The axial anomaly is responsible for the mass of the η' meson in the chiral limit

The fate of Chiral Symmetry in QCD



Chiral Perturbation Theory

Low energy effective theory for GBs and the QuasiGB η'.

$$\mathcal{L}_\chi = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \cdots$$
$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} Tr(\partial_\mu U \partial_\mu U^\dagger) + \frac{2B_0F_0^2}{4} Tr(\mathcal{M}_q U + h.c)$$
$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}_{GL} + \mathcal{L}^{(4)}_{WZW}$$

$$U = exp(\frac{i}{F_0}\pi^a\lambda_a)$$

$\mathcal{L}^{(4)}_{WZW}$ provides the anomaly matching for chiral anomalies: coefficients fixed!

To include η_1 explicitly need $1/N_c$ expansion: count $1/N_c$ on same footing as chiral counting order p^2
Pascual et al; Kaiser & Leutwyler

Singlet η

η_1 acquires mass because of axial anomaly. In chiral limit:

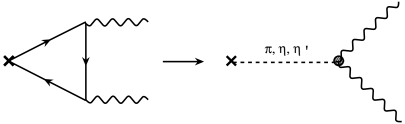
$$M^2_{\eta_1} = M_0^2 = \frac{4N_f}{F_0^2}\,\chi$$
$$\chi = \left(\frac{\alpha_s}{4\pi}\right)^2 < G\tilde{G}\ G\tilde{G} >= \mathcal{O}(N_c^0)$$

Topological susceptibility χ has been evaluated in lattice
Smith & Teper

$$M^2_{\eta_1} = \mathcal{O}(N_f/N_c)$$

In large N_c η_1 becomes GB: what happens at $N_c = 3$?

Matching anomalies



$= F_0 \; p^\mu \frac{1}{p^2 - M^2} \; \kappa \; F^{\mu\nu} \tilde{F}_{\mu\nu}$

κ fixed by matching anomalies in chiral limit: contained in WZW term

This gives chiral limit predictions for the amplitudes
$\pi^0 = \pi_3 \to \gamma\gamma$, $\eta_8 \to \gamma\gamma$ and $\eta_1 \to \gamma\gamma$

$$A(\pi_a \to \gamma\gamma) = i\alpha \frac{N_c}{12\pi} \frac{C_a}{F_0} F\tilde{F}$$

$$C_3 = 1, \; C_8 = 1/\sqrt{3}, \; C_0 = \sqrt{8/3}$$

π^0 width consistent with PDG values: precise test of anomaly
No anomaly: width would be $\sim 10^{-4}$ times smaller!
Bad for η and η'
Chiral symmetry breaking must be important

Chiral symmetry breaking

Quark masses break chiral and $SU(3)$ symmetries

Two main sources of corrections to $\gamma\gamma$ amplitudes

- State mixing:

$$\begin{aligned}\pi^0 &= (1-\epsilon^2-\tilde{\epsilon}^2)\,\pi_3+\epsilon\,\eta_8+\tilde{\epsilon}\,\eta_1\\ \eta &= \cos\theta\,\eta_8-\sin\theta\,\eta_1\\ \eta' &= \cos\theta\,\eta_1+\sin\theta\,\eta_8\end{aligned}$$

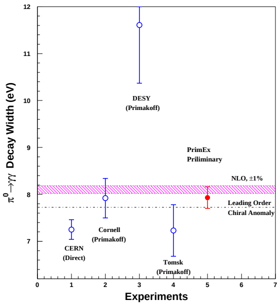
- Decay constants

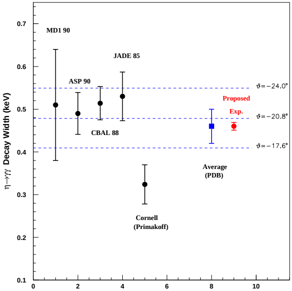
Order of mixings

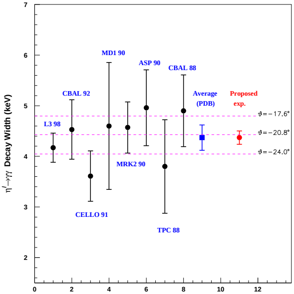
$$\epsilon=\mathcal{O}(\frac{m_d-m_u}{m_s})\qquad \tilde{\epsilon}=\mathcal{O}(\frac{B_0(m_d-m_u)}{M_0^2})=\mathcal{O}(p^2\,N_c)$$

$$\tan\theta=\mathcal{O}(\frac{B_0 m_s}{M_0^2})$$

$\pi^0,\ \eta,\ \eta' \to 2\gamma$: Empirical status







PRIMEX: more in Ashot's talk

Decays @ LO

Widths predicted in terms of F_0, M_0, and quark mass ratios $2m_s/(m_u+m_d)$ and $R = m_s/(m_d-m_u)$

$\Gamma_{\gamma\gamma}$	LO	No Mixing	Exp. PDG Avg.
π^0	8.08 eV	7.73 eV	7.7 ± 0.6 eV
η	613 eV	170 eV	464 ± 45 eV
η'	4.86 keV	7.36 keV	4.3 ± 0.4 keV

$$\epsilon = 1^o, \quad \tilde{\epsilon} = 0.5^o, \quad \theta = -20^o, \quad M_0 = 850\ MeV$$
$$2m_s/(m_u+m_d) = 26, \quad R = 45 \tag{1}$$

Main effects correctly described at LO:

- Mixing is essential in η and η' decays: good agreement at LO.
- π^0 width enhanced by 5% as result of mixing: mixing with η_8 alone gives $\sim 3\%$ enhancement.
  Moussallam; Bernstein, JG & Holstein

Decays @ NLO

Bernstein, JG & Holstein; Ananthanarayan & Moussallam

NLO corrections needed:

- Corrections to decay constants
- Better fit to masses: η mass is 50 MeV too low at LO
- Test stability of enhancement of $\Gamma_{\pi^0\to\gamma\gamma}$ (NLO corrections $\propto m_s$)

NLO analysis

- Inputs: $\Gamma_{\eta\to\gamma\gamma}$, $\Gamma_{\eta'\to\gamma\gamma}$, quark mass ratios (using NLO corrections to Dashen's theorem), LECs: L_5 and L_8
- Fit parameters: LECs: M_0, Λ_1, Λ_2
- Estimated inputs (cannot be fitted): LECs: t_1, K_1 (insignificant for π^0, but possibly relevant for η')

NLO results $\pi^0 \to \gamma\gamma$

	ϵ	$= 0.85 \pm 0.10^o$
	$\tilde{\epsilon}$	$= 0.30 \pm 0.03^o$
	θ	$= -10 \pm 2^o$
	$\Gamma_{\pi^0 \to \gamma\gamma}$	$= 8.12 \pm 0.07$ eV
With EM corr.	$\Gamma_{\pi^0 \to \gamma\gamma}$	$= 8.08 \pm 0.07$ eV

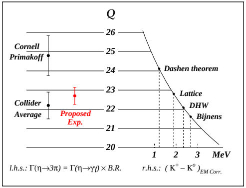
EM corrections Ananthanarayan & Moussallam: only enter through corrections to decay constants: F_{π^+} vs F_{π^0}.

Uncertainties in F_{π^+}, R and t_1 $\to$ theoretical error is $< 1\%$.

η

Partial decay widths of η normalized to $\Gamma(\eta \to \gamma\gamma)$: $(BR_{\gamma\gamma} \sim 40\%)$
Empirical improvement of $\Gamma(\eta \to \gamma\gamma)$ needed
$e^+ - e^-$ experiments consistent, but inconsistent with Primakoff (Cornell)

Important witdth: $\eta \to 3\pi$



$$Q^2 = \frac{m_s^2 - (m_u + m_d)/2}{m_d^2 - m_u^2}$$

Leutwyler

- Decay amplitude $\propto (m_d - m_s)$
- Needs NLO χPT (also done at NNLO!)
- $\pi\pi$ FSI: analyzed using dispersion relations
- Small EM corrections
- Competitive or even better way of obtaining ratio R

η'

Important question: how much of the GB qualities are kept by η'?
A list of suggestive facts:

- Witten's mass formula vis-á-vis determination of M_0:

$$LO: \quad M_0 = 850 MeV \implies \chi = (0.151\ \mathrm{GeV})^4$$
$$\chi_{\mathrm{Lattice}} = (0.187 \pm 0.022\ \mathrm{GeV})^4 \qquad \text{Smith \& Teper}$$

- LO $\eta - \eta'$ mixing predicted from quark mass ratio $m_s/(m_u + m_d)$, receives large NLO correction
- Coupling $\eta_1 - \gamma\gamma$ from WZW term $\oplus$ mixing: widths OK with exp.
- $F_{\eta'}$ similar to F_π: consistent with chiral and $1/N_c$ counting
- Precision $\pi^0 \to \gamma\gamma$ width (1.5%) would show relevance or not of $\pi^0 - \eta'$ mixing

$\eta - \eta'$ Mixing

- From the $\gamma\gamma$ decays: $\theta = -20^o$ at LO and -10 ± 2^o at NLO. Both $1/N_c$ and the corrections $\mathcal{O}(m_s)$ add up to big correction

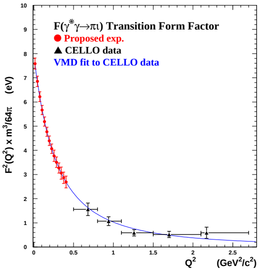
- Other analyses: Feldman et al.; Escribano & Frère

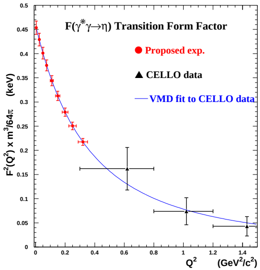
$$\frac{\Gamma(J/\psi \to \eta'\gamma)}{(J/\psi \to \eta\gamma)} \quad \to \theta = -15 \pm 3^o$$
$$\frac{\Gamma(J/\psi \to \eta' V)}{(J/\psi \to \eta V)} \quad \to \theta = -17 \pm 2^o$$

$SU(3)$ breaking in amplitudes not included in these analyses.

Form Factors

Experimental status





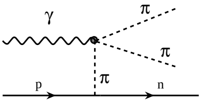
Radii determined by one LEC of $\mathcal{L}^{(6)}$:

$$t_2 = \begin{cases} \dfrac{3}{12\pi^2\Lambda^2} & \text{VMD} \\[2ex] \dfrac{3}{192\pi^2\tilde{m}_q^2} & \text{NJL} \end{cases}$$

Dipole fits $\Lambda_{\pi^0} = 0.75 \pm 0.03\text{GeV}, \quad \Lambda_\eta = 0.65 - 0.90\text{GeV}$

$\gamma p \rightarrow \pi^+ \pi^0 n$

Important process mediated by WZW term



$$A(\gamma\pi^+ \rightarrow \pi^+\pi^0) = -iF^{3\pi}\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu p_{1\nu} p_{2\rho} p_{3\sigma}$$

$$F^{3\pi} = \frac{eN_c}{12\pi^2 F_\pi^3} + \mathcal{O}(p^2) = 9.72\ \mathrm{GeV}^{-3} + \cdots$$

$$F^{3\pi}_{\mathrm{Exp}} = 10.7 \pm 1.2\ \mathrm{GeV}^{-3} \quad \mathrm{Serpukhov(1987)}$$

Hall B Miskimen et al; no final analysis published

$\eta \to \pi^0\gamma\gamma$

$\mathcal{O}(p^6) = \mathcal{O}(m_d - m_u) \times \mathcal{O}(p^4)$: very suppressed.
Experimental values kept changing: newest from Crystal Ball and KLOE:

$$\Gamma = 0.29 \pm 0.06 \pm 0.02 \text{ eV} \qquad \text{MAMI}$$
$$\Gamma = 0.11 \pm 0.04 \pm 0.02 \text{ eV} \qquad \text{KLOE (Prelim.)}$$

Important corrections (unitarization) beyond $\mathcal{O}(p^6)$ χPT
Oset&Pelaez

The whole story in Liping's talk

Summary

- $\pi^0 \to \gamma\gamma$ has tested chiral anomaly
- Increased precision by PRIMEX exposes quark mass effects: mixing with η and perhaps η'. Theory predicts 5% enhancement
- η partial rates are normalized to the $\gamma\gamma$ rates
- Need Primakoff for $\eta \to \gamma\gamma$ to cross check $e^+ - e^-$ results
- Improve $\Gamma(\eta \to 3\pi)$ to have better determination of $R = m_s/(m_d - m_u)$
- Still understanding what the GB traits of η' are
- Other interesting processes with η and η' may be accessible in HALL D